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KINEMATIC INTERPRETATION NON-UNDERSHUTTING CONDITIONS OF THE ROTORS OF ROTARY LOBE PUMPS AND GAS BLOWERS TYPE ROOTS

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Abstract One of the most important problems in the theory of pump rotor coupling is the study of profile coupling and the necessary and sufficient conditions that prevent rotor profile wedging. Essentially, the profile coupling concept represents local conditions that apply only to the point of contact, while not being provided for any other points. Rotors do not contact each other in any position, but the profils can lead to mutual engagement at some point, causing undercutting and wedging of the rotor. The traditional method for determining the non-undercutting condition of rotors is presented by Colbourne's studies. This method is based on geometrical considerations of the ratio of cases of special examples. Therefore, this procedure is general in relation to all other traditional methods of studying non-undercutting conditions. In this case, it is sufficient to consider the methods developed for spiral profiles, which are sometimes also based on graphical methods. At a later stage, advantages appear that are easily explained, but not usable, especially in parametric design.

Keywords: Rotary lobe positive displacement pump; kinematic interpretation; non-undercutting of rotor; limit curve; parametric design; rotor analytical model; coupled single-lobe; two-lobe and tri-lobe rotor profiles.

1. INTRODUCTION

In contrast to the traditional method presented by Colbourne's studies, the method presented in the paper [1], which is based on Litvin's theory, is accurate according to its analytical formulation on the one hand, and can be represented graphically on the other hand. Even more, since this method is easily solved by software, it enables the parametric design of the rotor profile and its verification.

The kinematic analysis used in this paper represents the method of determining and the trajectory of the limit curve. The limit curve concept defined by Litvin allows not only the presentation of the wedging of the coupled rotor profiles, but also the accurate determination of the lobe points of the rotor on the driven shaft caused by the undercutting of the rotor on the driving shaft [2-14].

2. KINEMATIC ANALYSIS

In this regard, according to the analysis of the kinematic pair of coupled profiles [1], where the radii of the rotor steps are , the following three equations can be written for rotary lobe positive displacement pumps and gas blowers type Roots:

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$$-d\sin\theta + 3a\sin(\varphi_2 + \theta) = 0 \tag{1}$$

$$r_{l2}3a\cos\theta\sin(\varphi_2 + \theta) + 2a\sin\varphi_2[-d\cos\theta + 3a\cos(\varphi_2 + \theta)] = 0 \tag{2}$$

$$r_{l2}3a\cos\theta\cos(\varphi_2 + \theta) + 2a\cos\varphi_2[-d\cos\theta + 3a\cos(\varphi_2 + \theta)] = 0 \tag{3}$$

(index 1 refers to the rotor on the driving shaft, and index 2 to the rotor on the driven shaft).

The unknown quantities in these equations are:
r_{l2} - the radius of the circle whose arc defines the profile of the lobe
θ - parameter,
where is φ_2 - the angle of rotation.
Size d - the distance between the center of the rotor on the driven shaft and the center of the circle whose arc defines the profile of the rotor, Figures 1, 2, 3, is known and pre-defined by the geometry of the rotor.

Equations (2) and (3) are not functions of the angular velocity, and are solved as functions of r_{l2} for each value of φ_2 and θ. Values from $\bar{r}_{l2}(\theta,\varphi_2)$ obtained from Eq. (2), also satisfy Eq. (3). It should be emphasized that equation (1) shows that the mutual engagement of the rotors is not a function of r_{l2}. This can be explained by considering the property of the circular profile, that the normals at the corresponding points of similar circles are definitely radial and independent of the radius of the circle. Also, equation (1) shows the mutual relationship between the angle of rotation φ_2 of the profile and the parameter θ, thus defining the coordinates of the points of contact between the path of the profile Γ_2 and the coupled profile Γ_1. Therefore, the maximum value of the radius of the lobe r_{l2} that does not lead to the undercutting profile Γ_1 represents the minimum value of the solution of equations (2) and (3).

The limit curve Γ_l, which represents the geometric locus of points that can cause wedging of coupled rotors, is defined as:

$$\Gamma_l^{(2)}:\begin{cases} x_{l2}^{(2)} = \bar{r}_{l2}(\theta,\varphi_2)\cos\theta + d \\ y_{l2}^{(2)} = \bar{r}_{l2}(\theta,\varphi_2)\sin\theta \end{cases} \tag{4}$$

and is shown graphically in the Figure together with the profile of the rotor on the driven shaft.

It should be said that the limit curve is not a function of the radius of curvature of the rotor lobe on the driven shaft, but depends on other design parameters. Therefore, if the drawn limit curve Γ_l does not intersect Γ_2, then the rotor profile Γ_1 does not cause undercutting. Otherwise, it is necessary to correct the radii of the arcs of the rotor profile.

The curve Γ_l should be graphically displayed as many times as the rotor on the driven shaft has lobes with a rotation of $\frac{2\pi}{i}$, where is i - the number of rotor lobes.

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3. RESULTS

Therefore, the kinematic analysis presented in this paper includes: (1) analytical model of the rotor, (2) analytical definition of the limit curve and (3) geometric interpretation of the obtained results based on examples from technical practice.

In this sense, the graphic representation of the geometrical interpretation of tri-lobe rotors of rotory lobe pumps for previously derived analytical models of rotors and limit curves and data from Table 1 is shown in Figure 4.

The graphic representation of the geometrical interpretation of two-lobe rotors of rotary lobe pumps for the previously derived analytical models of rotors and limit curves and data from Table 2 is shown in Figure 5.

The graphic representation of the geometrical interpretation of single-lobe rotors of rotary-lobe pumps for previously derived analytical models of rotors and limit curves and data from Table 3 is shown in Figure 6.

Table 1. Geometrical parameters of SLOMAN tri-lobe rotary lobe pumps.

i		3
$2a$	mm	60.93
r	mm	15.44
d	mm	31.65

Table 2. Geometrical parameters of SLOMAN two-lobe rotary lobe pumps.

i		2
$2a$	mm	60.93
r	mm	15.83
d	mm	23.26

Table 3. Geometrical parameters of SLOMAN single-lobe rotary lobe pumps.

i		1
$2a$	mm	60.93
r	mm	15.83
d	mm	23.39

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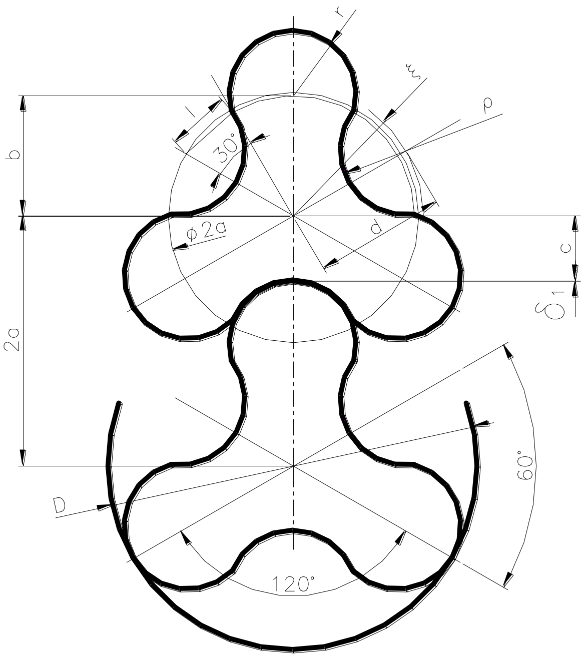

Figure 1. Geometrical parameters of tri-lobe rotors (i=3).

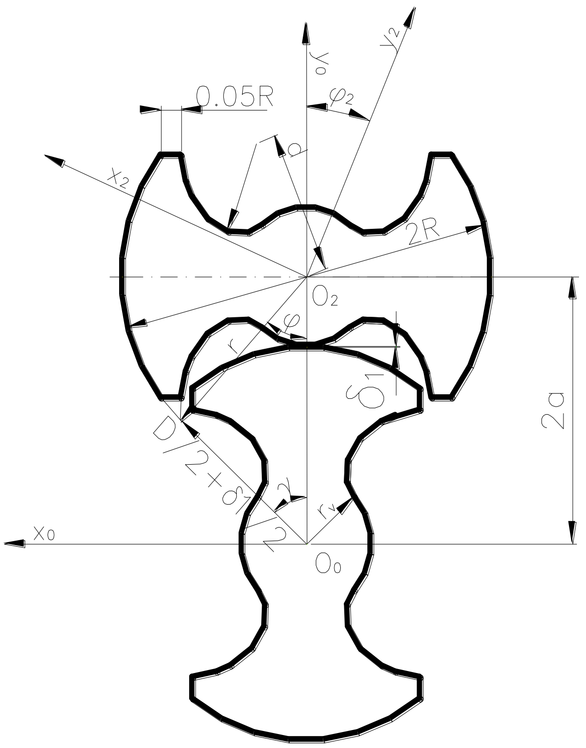

Figure 2. Geometrical parameters of two-lobe rotors (i=2).

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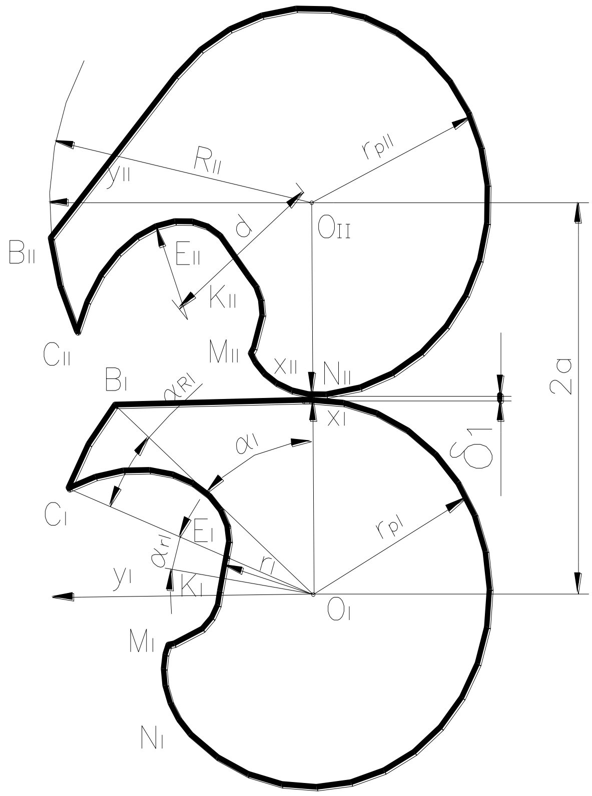

Figure 3. Geometrical parameters of single-lobe rotors (i=1).

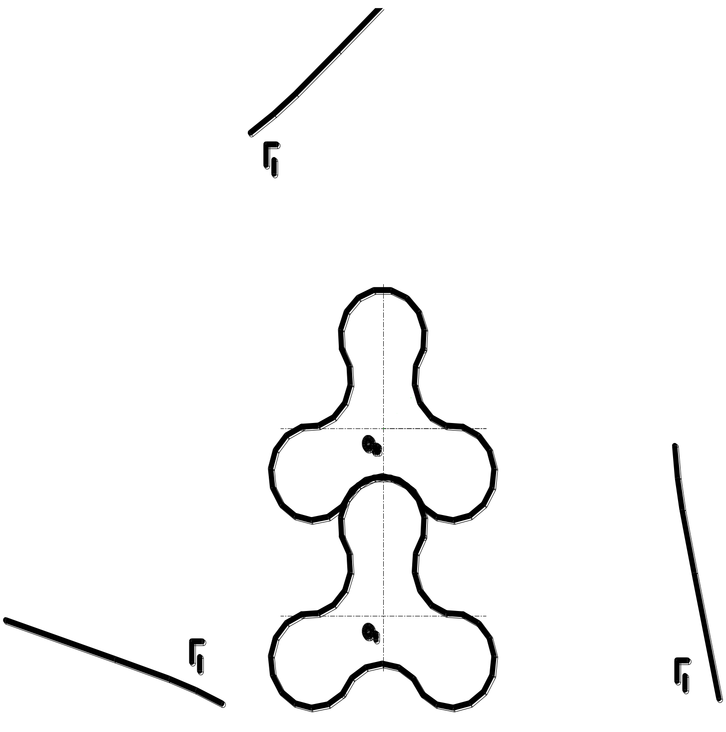

Figure 4. Geometric interpretation of tri-lobe rotors of SLOMAN rotary lobe pumps.

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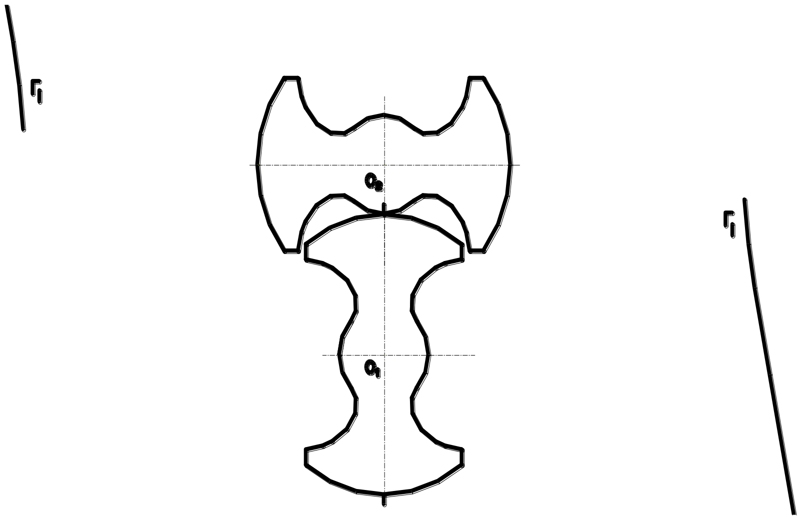

Figure 5. Geometric interpretation of two-lobe rotors of SLOMAN rotary lobe pumps.

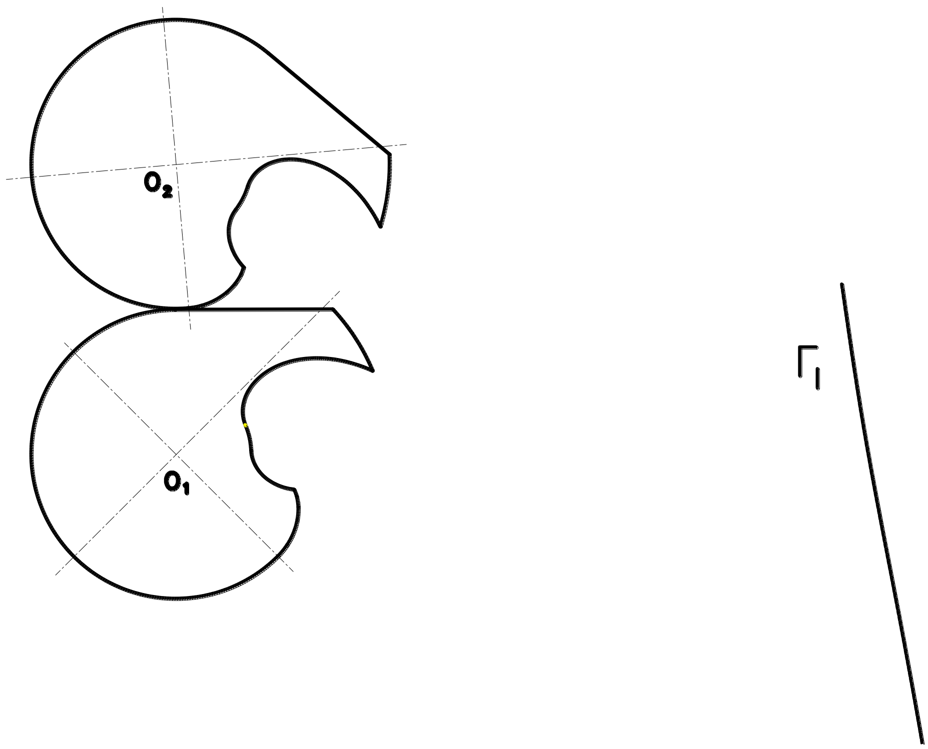

Figure 6. Geometric interpretation of single-lobe rotors of SLOMAN rotary lobe pumps.

All three previous figures, therefore, show that the limit curve does not intersect or have points of contact with the rotor profile Γ_2 on the driven shaft, so the rotor profile Γ_1 does not cause undercutting. This proves that the rotor profiles are correctly modeled, that no modifications to the profile geometry are required and that the effective design of the rotor prevents any wedging of the rotors.

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The distances of the rotor profiles from the limit curve Γ_l show that the smallest clearances between the coupled rotor profiles are achieved with tri-lobe rotors, the middle with two-lobe rotors, and the largest clearances with single-lobe rotors of rotory-lobe pumps and the gas blowers type Roots.

4. CONCLUSION

The necessity of achieving small clearances between the profiled surfaces of the rotor and the corresponding high accuracy of their production require the application of analytical methods for solving the task. Graphical methods cannot provide either the required accuracy or the necessary uniformity. In this paper, the selected domains, among several offered, prevent the existence of a working clearance between the corner point of one of the rotors and the lobe of the other in all appropriate positions, because the point of greatest approximation moves along the entire deflection profile and local machining inaccuracies or crushing during accidental impact do not significantly affect the volumetric losses. At the same time, only two of those proportions - half of the axis distance of the rotors and the radius of the rotor housing or half of the axis distance of the rotors and the distance of the center of protrusion of the profile lobe from the axis of the rotor-can be set arbitrarily.

In this work, the applied universal method of kinematic interpretation of the conditions of rotor non-undercutting is not limited only to specific and special cases of rotor geometry of rotary-lobe pumps and gas blowers type Roots. On the contrary, the presented method is general and shows the analytical conditions for avoiding undercutting of the rotors of displacement pumps and gas blowers by using the concept of the limit curve. This enabled the analytical definition of the limit curve, which graphically indicates the design limitations and, in this sense, the advantage of using tri-lobe rotors.

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